Determination of

PEAK DISCHARGE-FREQUENCY RELATIONSHIPS

for streams within a selected area in California

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DETERMINATION OF PEAK DISCHARGE-FREQUENCY RELATIONSHIPS FOR STREAMS WITHIN A SELECTED AREA IN CALIFORNIA

Earl Lock Neff and Paul C. Sheffer[1]

INTRODUCTION

One of the more common problems facing an engineer or hydrologist is that of finding discharge-frequency relationships for areas in which there are no adequate stream-gaging records available. There are several methods in use by which frequency curves can be synthesized. Among these are the so-called "rational method," unit hydrograph procedures, and regional stream studies. Each of these, in one way or another, takes into consideration climatic and physiographic factors which influence peak rates of runoff. This report presents the results of a regional stream study conducted using stream records from U. S. Geological Survey gaged watersheds on the western slopes of the Sierra Nevadas. The map, Figure 1, shows the general area from which data were taken.

Regional stream studies are confined to areas, called hydrologic zones or provinces, in which there is a similarity in the following climatic and physiographic characteristics:

<u>Precipitation</u>
- Distribution through the year
- Amounts and rates
- Form - whether rain or snow

<u>Topography</u>
- Elevation
- Land and channel slopes
- Watershed shape

<u>Geology</u>

<u>Soils</u>

Each of these factors is evaluated graphically or statistically to determine its influence on frequency curves of annual peak discharge from recorded data within a hydrologic zone. The results of such an analysis can then be used to estimate frequency curves for ungaged watersheds that lie within the same hydrologic province.

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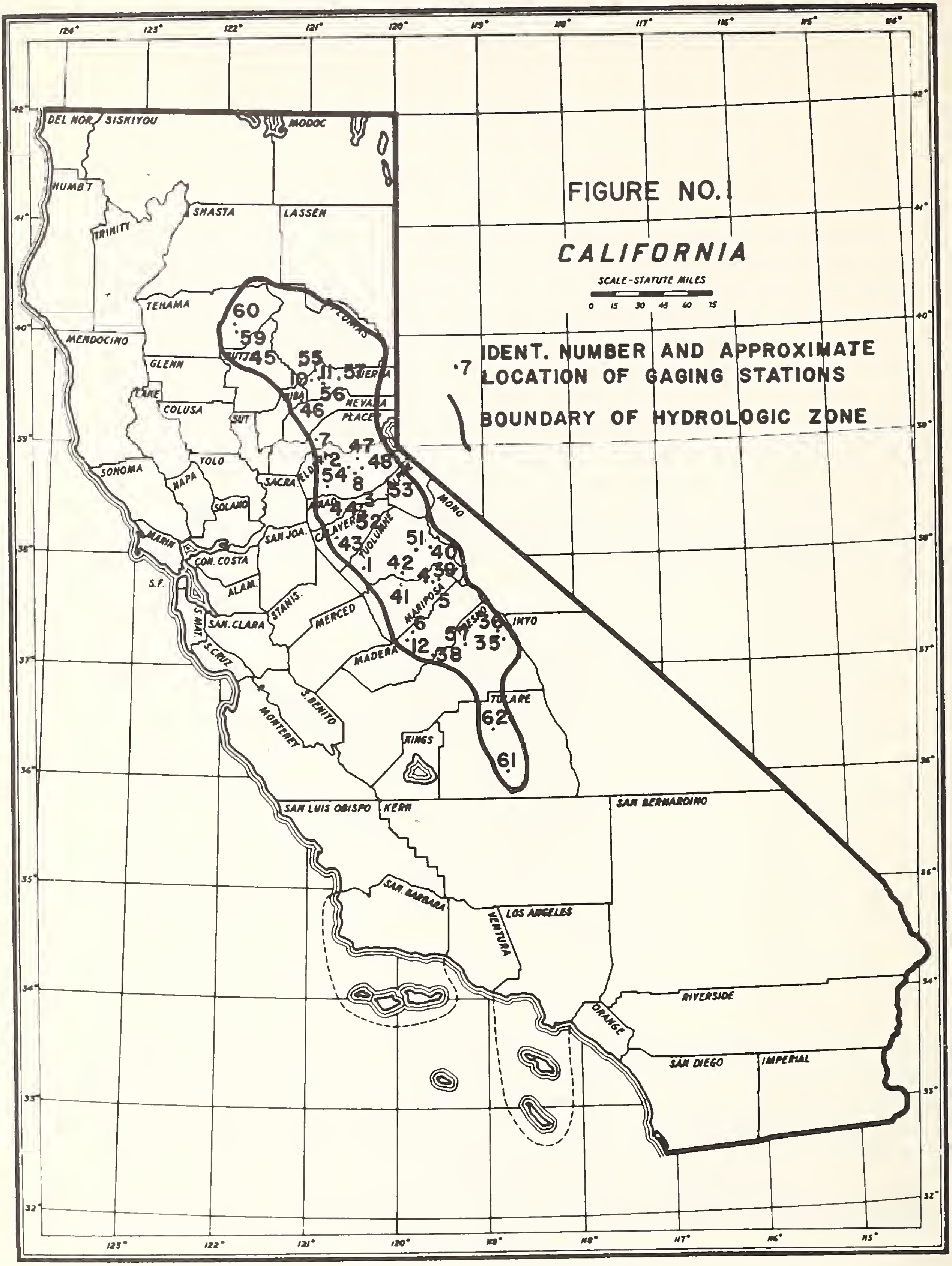


FIGURE NO. I
CALIFORNIA
SCALE-STATUTE MILES
0 15 30 45 60 75
IDENT. NUMBER AND APPROXIMATE LOCATION OF GAGING STATIONS
BOUNDARY OF HYDROLOGIC ZONE
DEL NOR
SISKIYOU
MODOC
HUMBT
TRINITY
SHASTA
LASSEN
TEHAMA
PLUMAS
MENDOCINO
GLENN
LAKE
COLUSA
SUT
NEVADA
PLACER
YUBA
SONOMA
YOLO
NAPA
SOLANO
SACRA.
MARIN
CON. COSTA
SAN JOA.
CALAVERAS
TUOLUMNE
MONO
S.F.
ALAM.
STANIS.
MERCED
MARIPOSA
S. MAT.
SAN. CLARA
S. CRUZ
MADERA
INYO
TULARE
MONTEREY
S. BENITO
KINGS
SAN LUIS OBISPO
KERN
SAN BERNARDINO
SAN. BARBARA
VENTURA
LOS ANGELES
RIVERSIDE
ORANGE
SAN DIEGO
IMPERIAL

JAN 1, 1925

If the theory of normal distribution of logarithms of hydrologic data is assumed, then the discharge-frequency relationship can be expressed as a straight line on logarithmic-normal graph paper. There are several ways by which it is possible to construct this straight line. One method is to use recorded data to compute the mean and the standard deviation of the logarithms of a series of discharges to arrive at the statistically most probable frequency line, while another method is to define the line by the mean event and the ratio of some other frequency event to the mean. In this study standard multiple regression procedures were used on the various climatic and physiographic parameters to determine equations for estimating the mean event and the ratio of the 100-year frequency event to the mean.

This report is presented in five parts: (1) the introduction, (2) parameters and symbols, (3) methods, (4) results, and (5) example and conclusion.

PARAMETERS AND SYMBOLS

The following watershed characteristics were considered during the course of this study, and each was evaluated to find its relative significance or influence on frequency curves of annual peak discharge:

1. Average annual precipitation. Data from nearby precipitation gages, published by the U. S. Weather Bureau in "Climatological Data", was used for each watershed.

2. Difference in elevation. The difference in elevation between the highest point and the outlet of the watershed was used to define an index of land and channel slopes. These values were taken from contour maps of a scale of 1:500,000.

3. Shape. A shape factor for each watershed was found by measuring the length of the longest straight line that could be drawn from the outlet to the watershed boundary and dividing this length by the length of the perpendicular bisector of that line contained within the watershed boundaries.

4. Drainage area. The drainage area in square miles, for each watershed, was taken from U. S. Geological Survey Water Supply Papers.

5. Mean watershed elevation. The mean elevation was defined as that elevation above which lies one-half of the drainage area of the watershed, and below which lies the other half. This value was measured by planimeter from the contour maps of a scale of 1:500,000.

6. Average annual runoff. The average annual runoff was found in U. S. Geological Survey Water Supply Papers and was expressed in units of inches for this study. Also "annual" refers to the water year, October through September.

The characteristics of the discharge-frequency lines that were considered are:

1. The 2-year frequency discharge (the 50% chance event).
2. The 100-year frequency discharge (the 1% chance event).
3. The ratio of the 100-year discharge to the 2-year discharge.

For easy reference, Table 1 has been prepared listing the various parameters along with the symbols and units used.

TABLE 1

Factor	Symbol	Units
Drainage area	A	Square miles
Elevation	E	Feet above mean sea level
Average annual runoff	Q_a	Inches
Average annual precipitation	P_a	Inches
Difference in elevation	H	Feet
Shape	S	$\frac{L}{W}$
2-year frequency discharge	q_2	cfs
100-year frequency discharge	q_{100}	cfs
Ratio of the 100-year event to the 2-year event	R	$\frac{q_{100}}{q_2}$

METHODS

The first step was to select watersheds in the study area that met the following basic criteria:

1. A minimum continuous record of approximately 20 years.
2. A maximum drainage area of 400 square miles.
3. Little or no regulation or diversion above the gaging station.

There were thirty-six watersheds in the area that would meet these specifications. These watersheds ranged in size from 7 square miles up to 321 square miles, with 22 of them having an area less than 100 square miles, 8 having an area between 100 and 200 square miles, and 6 having an area 200 to 400 square miles. The period of record ranged from 17 years up to 42 years. Of these, 4 had periods of record of 17 to 20 years, 16 had 20 to 30 years, 13 had 30 to 40 years, and 3 had periods of 40 to 42 years. The mean elevations of the waterhseds ranged from 900 ft. msl. to 10,700 ft. There were 5 watersheds with elevations between 900 and 3000 ft., 21 between 3000 and 7000 ft., 8 between 7000 and 9000 ft., and 2 between 9000 and 10,700 ft. The average annual runoff varied from 5.24 inches to 42.86 inches with 6 watersheds having average annual runoff of 5 to 10 inches, 9 having 10 to 20 inches, 10 having 20 to 30 inches, 8 having 30 to 40 inches, and 3 having over 40 inches.

Next, for each watershed the annual flood (maximum instantaneous peak discharge recorded during the year) and annual volume of runoff (total amount of water yield for the entire water year expressed in inches) was tabulated for each year of the period of record. From these data the average annual runoff (Q_a) was found by a straight arithmetic average of the annual volumes of runoff, and discharge-frequency lines were computed, using the annual floods, by the "Hazen" computing method[2].

The third step was the compilation and tabulation of the information listed above in the section "Parameters and Symbols". These data were found by measurement from available maps or by extraction from published records.

In the fourth step each watershed was assigned an identification number, and from a table of random numbers twenty-three were selected to be used in the multiple regression analysis. The remaining thirteen watersheds were held in reserve to be used as a check of the results of the analysis. Table 2 lists the gaging stations that were used in the regression analysis and Table 3 lists those held as checks.

The fifth, and final, step was the actual analysis itself, in which standard multiple regression techniques were used to evaluate the influence of each of the various parameters on frequency curve characteristics.

RESULTS

To be useful the results of any study of this kind must meet two criteria:

1. That they be fairly simple and easy to use by individuals who do not have an extensive background in hydrology or statistics.

2. That the factors or parameters used be readily available, either from maps or published records, and do not require extensive or detailed field surveys.

Each of the parameters used in this study will be discussed below, along with the reasons why each was considered and the results of using it in the regression analysis.

1. Average annual precipitation is one of the major factors influencing discharge frequencies. In most cases frequency lines for watersheds in arid or semi-arid areas are considerably steeper than are frequency lines for watersheds in more humid regions. Unfortunately, in the hydrologic zone of this study, the distribution of precipitation gages was such that it was not possible to determine average precipitation over the watersheds. As a result, the precipitation factor does not appear in the final regression equations.

2. The difference in elevation between the highest point and the outlet of the watershed did not prove to be significant in this study, although it might be important in studies concerning other hydrologic areas.

[2] Soil Conservation Service Engineering Handbook, Section 4, Snpplement A, "Hydrology," page 3.18 - 12.

TABLE 2

Gaging Stations Used in Multiple Regression Analysis

Gaging Station	Designation No.	Drainage Area (Sq. Mi.)	Mean Elevation (Ft.)	Average Annual Runoff (In.)	Years of Record	Difference in Elevation (Ft.)	Shape Factor
Woods Cr. nr. Jacksonville	1	98	1900	8.42	28	2500	2.9
Silver Cr. nr. Placerville	2	176	5600	29.94	33	3000	4.4
Merced R. at Pohono Br.	4	321	8350	25.26	38	3500	3.6
Fresno R. nr. Knowles	6	132	2900	8.62	38	3500	1.8
No. Fk. American R. nr. Colfax	7	308	4500	29.32	28	7500	4.2
No. Fk. Yuba R. at Goodyear Bar	10	214	6100	36.49	18	4000	1.4
Chowchilla R. at Buchanan Dam	12	238	1900	6.02	23	5000	1.8
Bear Cr. nr. Vermillion Valley	35	54	10700	21.76	31	3500	1.9
Pitman Cr. below Tamarack Cr.	37	22	7900	23.81	25	3500	1.7
Fine Gold Cr. nr. Friant	38	93	1900	5.96	17	1500	3.8
Tenay Cr. nr. Yosemite	39	47	8600	30.52	39	5500	1.7
Falls Cr. nr. Hetch Hetchy	40	45	8000	42.86	38	4000	14.7
So. Fk. Tuolumne R. nr. Oakland R. C.	41	88	5300	14.36	30	6000	4.0
Mid. Fk. Tuolumne R. nr. Oakland R. C.	42	71	6500	14.03	36	6000	14.8
Cosgrove Cr. nr. Valley Spr.	43	21	900	5.24	24	500	2.0
Silver Cr. at Union Valley	47	83	5900	35.12	30	4000	1.8
Cherry Cr. nr. Hetch Hetchy	51	111	7750	42.29	18	5000	3.9
So. Fk. Mokelumne R. nr. R. R. Flat	52	37	5100	17.16	21	3500	3.4
Goodyear Cr. at Goodyear Bar	55	12	4000	42.10	20	2500	2.0
Rock Cr. at Goodyear Bar	56	11	4500	30.81	20	3000	2.2
No. Fk. Yuba R. nr. Sierra City	57	91	6150	32.91	20	3000	1.8
So. Fk. Tule R. nr. Success	61	105	4250	5.54	24	6500	1.7
No. Fk. Kaweah R. at Kaweah	62	128	5300	10.88	41	7000	1.7

TABLE 3

Gaging Stations Used to Check Results of Regression Analysis

Mid. Fk. Mokelumne R. at West Point	3	67	4400	12.16	41	4500	7.6
Merced R. at Happy Isles Br.	5	181	8800	25.52	39	5000	1.8
Plum Cr. nr. Riverton	8	7	5000	15.46	17	1500	3.3
No. Fk. Yuba R. nr. Goodyear Bar	11	244	5900	39.49	23	4500	1.6
Mono Cr. nr. Vermillion Valley	36	92	10000	21.65	31	4500	3.1
So. Fk. Mokelumne R. nr. West Point	44	74	4000	15.75	20	3000	3.1
Chico Cr. nr. Chico	45	68	3300	28.62	22	5500	16.0
Oregon Cr. nr. No. San Juan	46	35	3850	29.96	42	4000	6.5
So. Fk. Silver Cr. nr. Ice House	48	28	6800	35.81	30	4500	5.2
Cold Cr. nr. Mokelumne Peak	53	23	7700	36.39	27	3500	3.0
No. Fk. Consumnes R. nr. El Dorado	54	197	3700	13.66	28	6500	3.9
Deer Cr. nr. Vina	59	200	4050	19.80	34	6000	5.6
Mill Cr. nr. Los Molinas	60	134	3000	28.78	26	7000	17.5

3. The shape of the watershed was another factor which did not prove to be significant in this study, but which might be important in studies of different hydrologic provinces.

4. As was expected, the drainage area was one of the important factors affecting the position or magnitude of discharge-frequency lines in this study. It is logical that with all other factors being equal a large area has a greater chance or probability of producing a discharge of any given size than does a smaller area. However, the drainage area did not prove to significantly affect the slope of the frequency lines. It was found that frequency lines from streams having similar watershed characteristics would have approximately the same slope in spite of large differences in size of the drainage areas.

5. Mean watershed elevation was another factor which proved significant in this study. While the physical difference in barometric pressure between watersheds with different elevations does not directly affect rates of runoff, the elevation can serve as an index of several watershed characteristics. Among these are:

   a. The form in which the precipitation comes. That is, whether it is rain or snow.
   b. To a certain extent, the amount of precipitation.
   c. The distribution of runoff-producing precipitation through the year.
   d. The distribution of temperature through the year.
   e. Land use, soils, and geology of the area.

   These items are important since the hydrographs that produce the peak flows each year from high elevation watersheds are usually the results of snow-melt and have a relatively low peak and a long duration. In contrast, the hydrographs from areas of lower elevation are more probably the result of a larger proportion of rainfall and have a higher peak and shorter duration.

6. Average annual runoff also proved to be significant in this study. While this parameter is more closely associated with peak rates than are some others, it is also indicative of:

   a. The amount of watershed precipitation.
   b. Land use and cover conditions of the area.
   c. Soils and geology of the area.

   Each of these factors, individually and in combination, affects the rates and volumes of the runoff hydrographs, and their influence can be indicated by the average annual runoff.

The final result of the analysis is a series of equations relating the watershed parameters to discharge-frequency line characteristics. It was found that dividing the watersheds studied into two groups gave more accurate results than when the watersheds

were considered as a single sample. Group I included the watersheds that had a mean elevation above 6500 feet msl., and Group II included those with elevations below 6500 feet. For both Group I and Group II regression equations were developed to estimate the 2-year frequency discharge (q_2) and the ratio (R) of the 100-year frequency discharge (q_{100}) to q_2. The analysis produced the following regression equations:

Group I - (Above 6500 ft. mean elevation)

$$q_2 = 597 \frac{A^{0.868} \, Q_a^{1.024}}{E^{0.694}} \qquad (1)$$

$$R = \frac{69{,}650}{E^{1.054} \, Q_a^{0.131}} \qquad (2)$$

Group II - (Below 6500 ft. mean elevation)

$$q_2 = 21{,}530 \frac{A^{0.979} \, Q_a^{0.766}}{E^{1.089}} \qquad (3)$$

$$R = 5.8 \frac{E^{0.184}}{Q_a^{0.459}} \qquad (4)$$

The results of applying the pertinent equations to the check watersheds of Table 3 are shown on the accompanying discharge-frequency graphs in the Appendix (p. 14). In each case the straight line was constructed by applying either equations (1) and (2) or equations (3) and (4) to the watershed characteristics of these areas and the points, indicated by circles, were plotted from the recorded data using the standard Hazen equation for plotting positions:

$$P = 100 \frac{2n-1}{2Y}$$

where P = plotting position in percent chance of occurrence, n = order number in the array of annual floods, and Y = number of years of record.

The curved lines are upper and lower confidence limits computed using the recorded data for each watershed and are plotted for comparison with the frequency line computed by use of the regression equations. There are nine chances out of ten (90% chance) that these limits will include the true discharge value for any given frequency. The position of the confidence limits depends upon the standard deviation and the number of items of recorded data. For data that have a large standard deviation (steep slope of the frequency line) the limits will be further apart than for data with a smaller standard deviation. Also, the limits will be further apart for short periods of record (few items) than they will be for longer periods.

Table 4 lists the check watersheds and provides an adjective description of the results of applying the derived equations to each. This description was made from visual examination of the frequency lines. Results were considered good if the straight line fell near the middle of the confidence limits, fair if it fell entirely within the confidence limits but close to either the upper or lower limit, and poor if it fell outside the confidence limits. This evaluation gave the following results:

TABLE 4

Comparison of Results of Applying Equations (1) and (2) or (3) and (4) to Synthesize Discharge-Frequency Lines

Watershed Designation Number	Rating
3	Poor
5	Good
8	Fair
11	Good
36	Poor
44	Good
45	Fair 10-50% chance; Good above 10%
46	Fair below 10%; Good above 10%
48	Fair below 10%; Good above 10%
53	Poor
54	Fair below 10%; Good above 10%
59	Good
60	Fair below 10%; Good above 10%

Summary of Table 4

Rating	Number	Percent
Good	4	31
Fair to Good	5	38
Fair	1	8
Poor	3	23

If these thirteen watersheds can be considered to constitute a true random sample, there are approximately three chances out of four that use of equations (1) and (2) or (3) and (4) will produce a discharge-frequency curve that is fair or better. And, conversely, there is one chance out of four that it will be poor.

The same comparison was made for the streams listed in Table 2. However, in order to keep illustrative material to a minimum, the discharge-frequency graphs for these streams are not included in this report. Instead only the adjective description of the results of this comparison are shown in Table 5.

TABLE 5

Comparison of Results of Applying Equations (1) and (2) or (3) and (4) to Synthesize Discharge-Frequency Lines

Watershed Designation Number	Rating
1	Good
2	Good
4	Good
6	Poor below 30% chance; Fair 10-30%; Good above 10%
7	Good
10	Good
12	Good
35	Fair
37	Poor
38	Good
39	Fair
40	Poor below 30%; Fair above 30%
41	Good
42	Fair
43	Fair below 50%; Good above 50%
47	Poor below 10%; Fair above 10%
51	Fair below 50%; Good above 50%
52	Good
55	Good
56	Good
57	Good
61	Good
62	Fair

Summary of Table 5

Rating	Number	Percent
Good	13	57
Fair to Good	2	9
Fair	4	17
Poor to Fair	2	9
Poor	1	4
Poor to Good	1	4

Eighty-three percent of these are fair or better for the entire frequency line and 96% are fair or better for values greater than the 10-year frequency (10% chance).

Another comparison that was made is shown on Figure 2, on which the 10-year frequency discharge as determined for each station by the Hazen computing method is plotted against the 10-year frequency discharge as determined by the regression equations derived in this study. Assuming all of the error to occur in the regression equations, the dashed lines indicate ±25% difference, while the solid line is one of equal values. There are three points that plot above the +25% line and two that plot below the −25% line. Three points that could not be plotted on this scale are not shown. The correlation coefficient for this comparison of the thirty-six watersheds is 0.97.

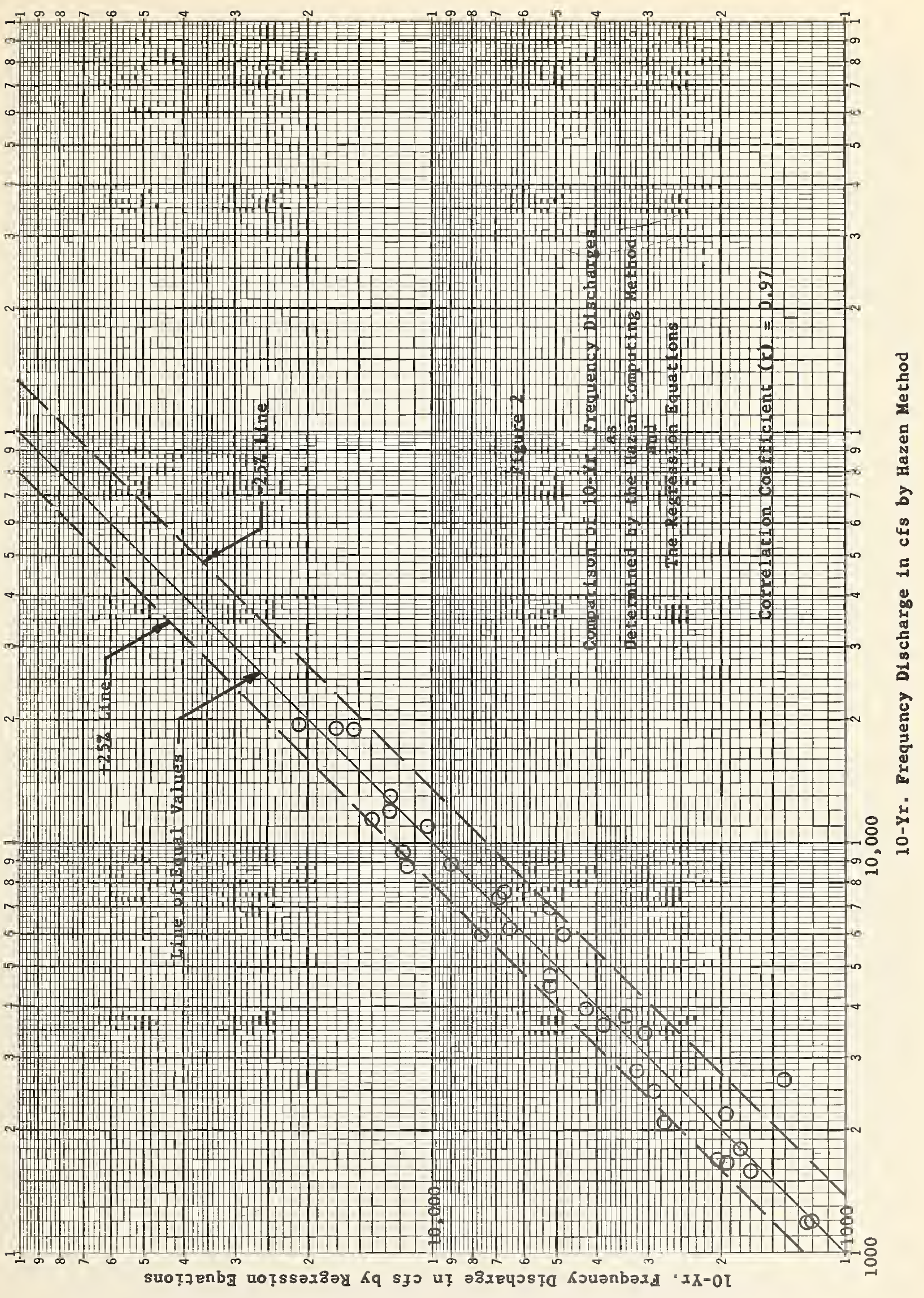


Figure 2

Comparison of 10-Yr. Frequency Discharges as Determined by the Hazen Computing Method and The Regression Equations

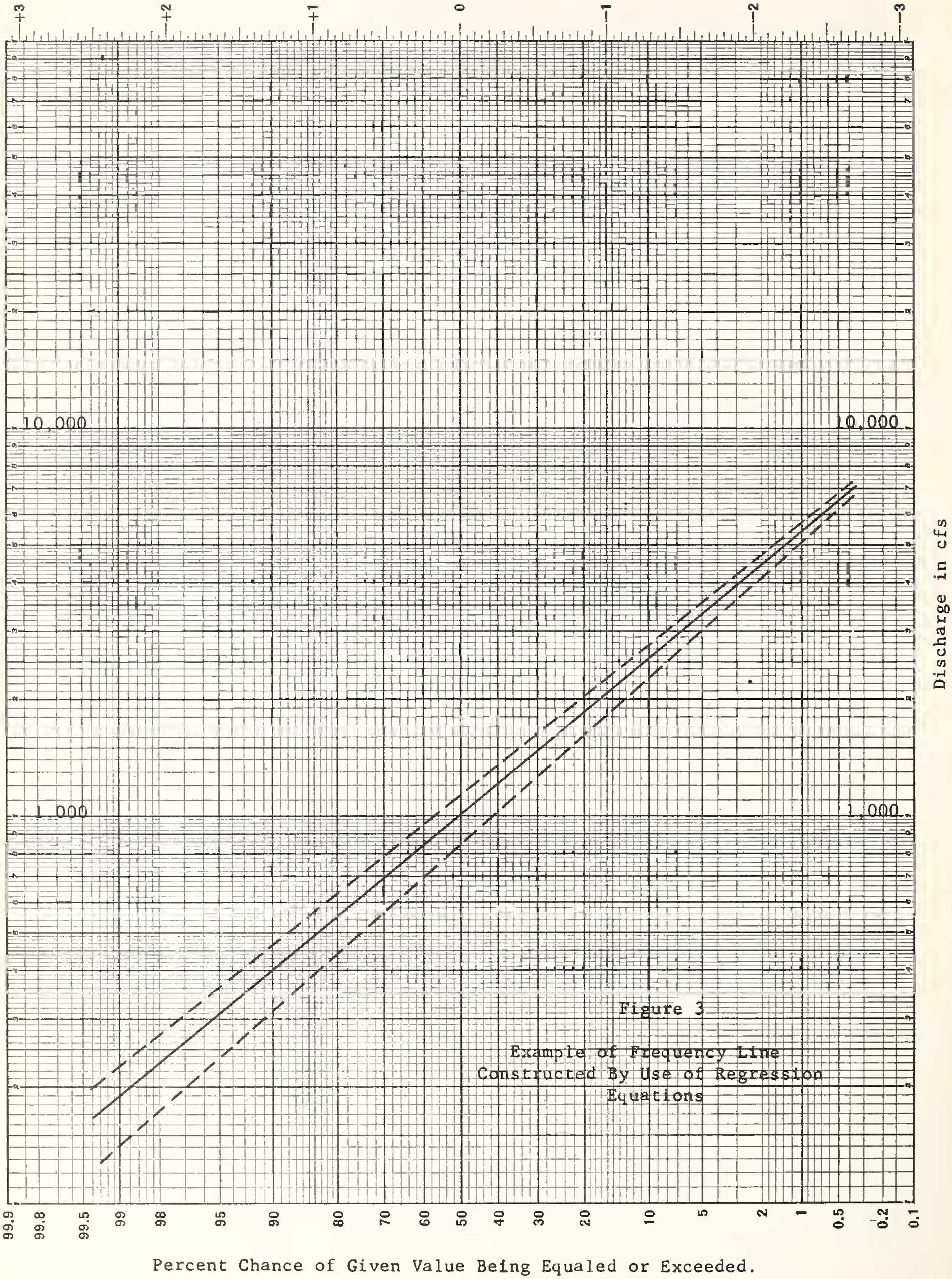


Figure 3

Example of Frequency Line Constructed By Use of Regression Equations

EXAMPLE AND CONCLUSION

The following example illustrates the application of the derived equations in estimating a discharge-frequency line for an ungaged watershed. Assume that it is desirable to develop a frequency line for a watershed with the following characteristics:

1. Location: Sierra County, Calif., near Goodyear Bar.
2. Drainage Area: 35.7 square miles as measured from a topographic map.
3. Mean Watershed Elevation: 5300 feet as measured from a topographic map.
4. Average Annual Runoff: 36.4 inches estimated by taking an arithmatic average of the recorded average annual runoff from:

   a. North Fork Yuba River at Goodyear Bar.
   b. North Fork Yuba River near Goodyear Bar.
   c. Goodyear Creek at Goodyear Bar.
   d. Rock Creek at Goodyear Bar.
   e. North Fork Yuba River near Sierra City.

These data are substituted into equations (3) and (4) with the following results:

1. q_2 = 1010 cfs
2. R = 5.4
3. q_{100} = (1010) x (5.4) = 5450 cfs

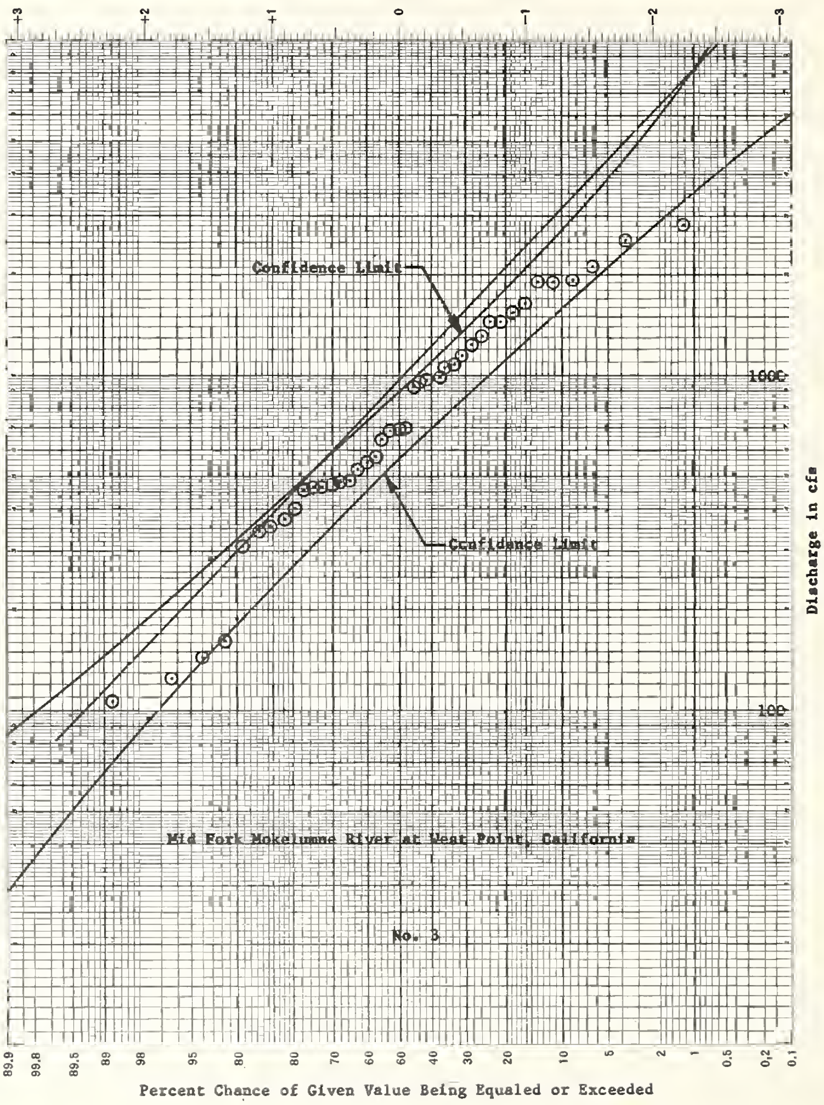
The q_2 value (1010 cfs) is plotted at the 50% chance, the q_{100} value (5450 cfs) at the 1% chance, and a frequency line is drawn through these two points as shown by the solid line in Figure 3.

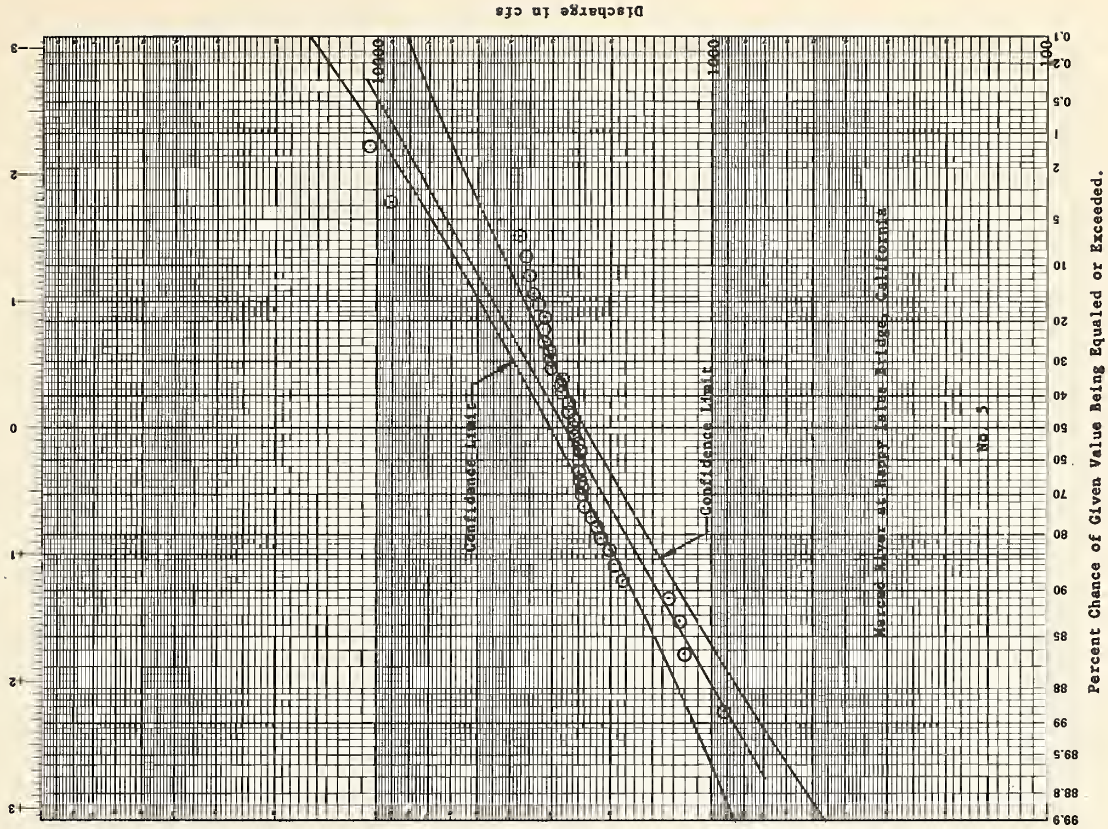
Each of the factors necessary for the construction of the frequency line can be determined with a fair degree of accuracy with the possible exception of the average annual runoff. For an ungaged watershed this value must be estimated by one method or another. The dashed lines of Figure 3 illustrate the effects of over- and under-estimating Q_a. The upper line was constructed using the same data as the above example except that Q_a was increased by 20% to 43.7 inches. The lower line was constructed using 29.1 inches, or a 20% reduction in Q_a. The 20% increase in Q_a, with all other factors remaining the same, produces an increase in q_2 of +12.9% and in q_{100} of 5.5%. A 20% decrease in Q_a produces a change in q_2 of −15.8% and in q_{100} of −6.4%. From this example it can be seen that the effects of over- or under-estimating Q_a are minimized in the final answer when equations (3) and (4) are used. However, for watersheds in which equations (1) and (2) are used a ±20% error in estimating Q_a will produce approximately ± 20% difference in both q_2 and q_{100} because of the exponents associated with Q_a in these equations.

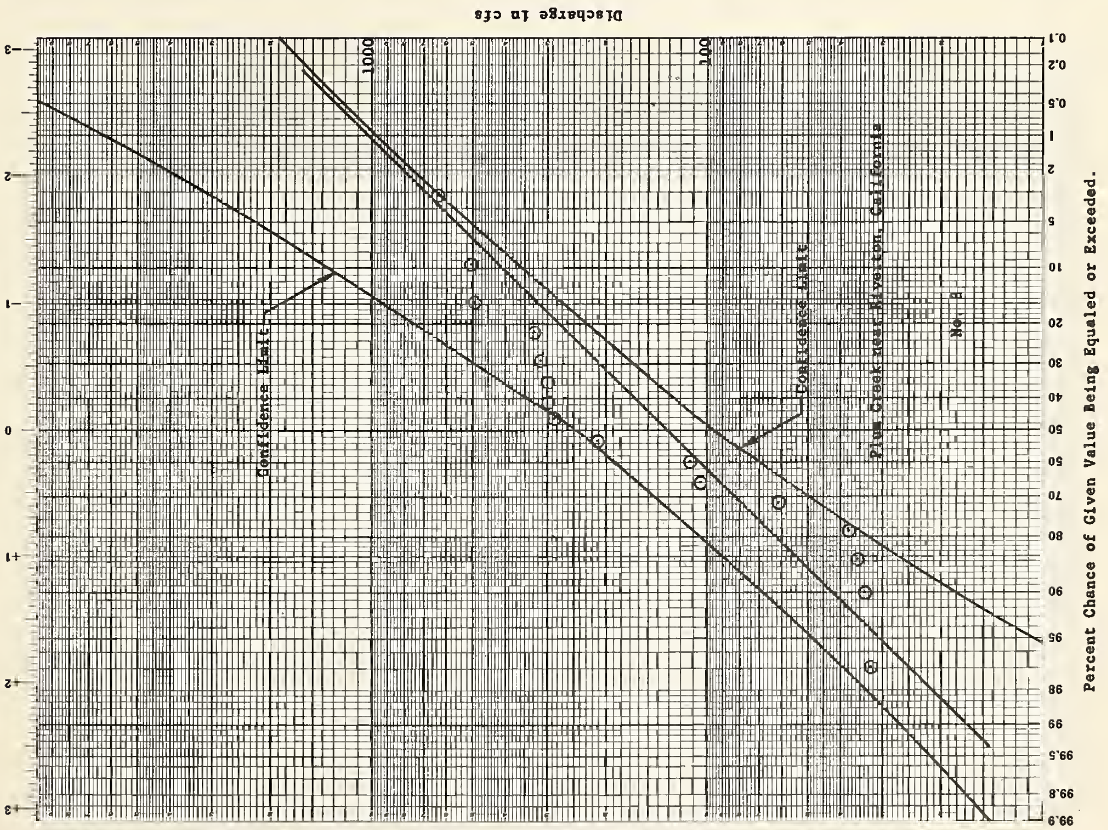
In conclusion, the results of this study are not as accurate as might be desirable. However, they are adequate for certain simple design problems or for preliminary investigations.

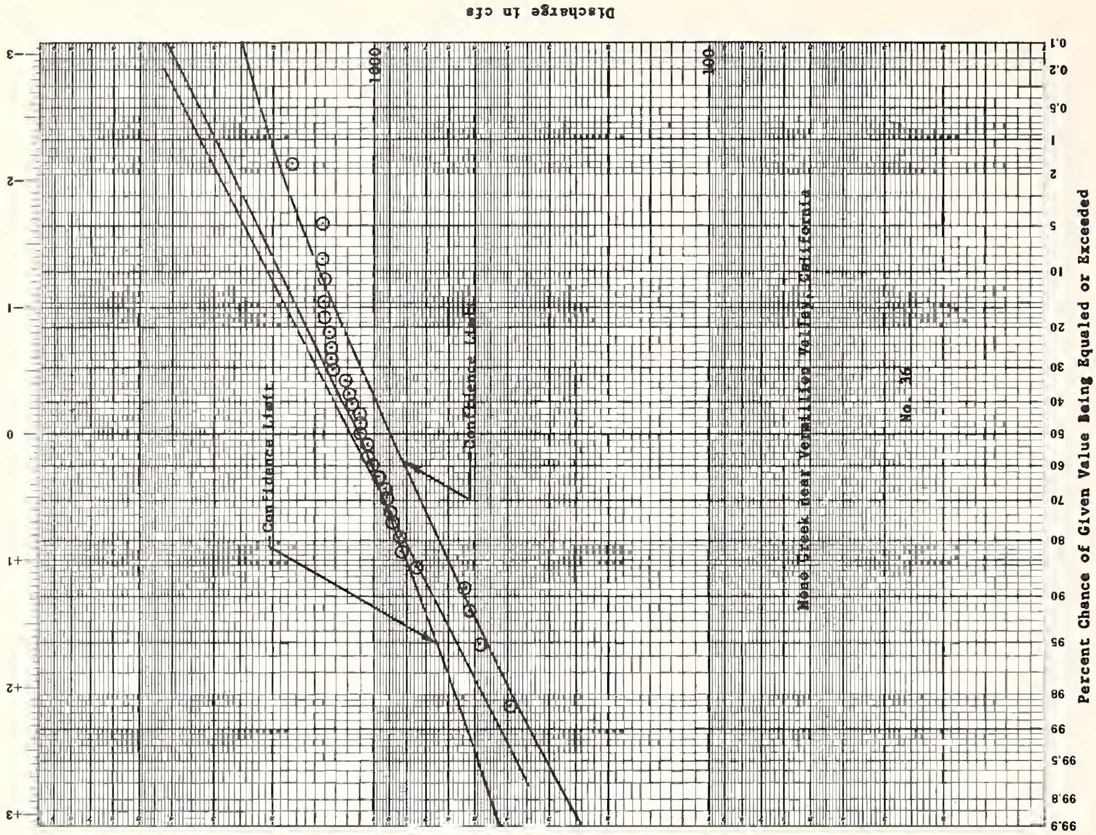
APPENDIX

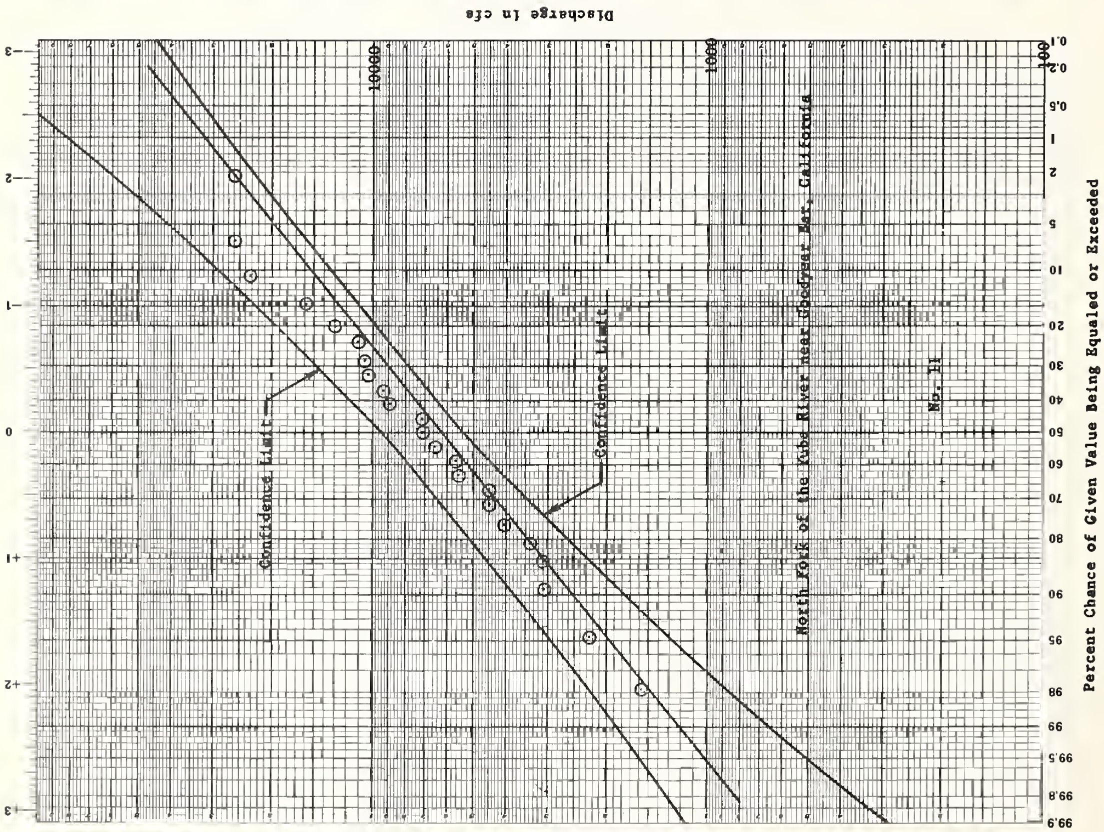
(Discharge-Frequency Graphs)

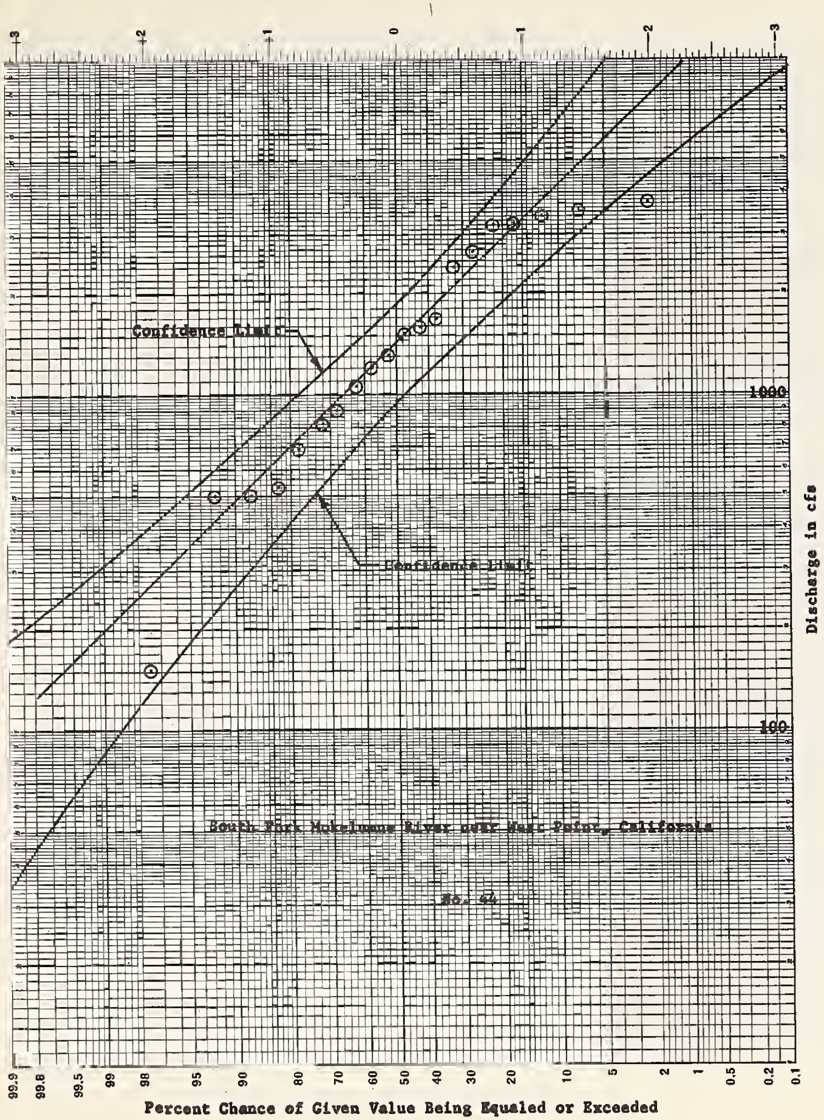


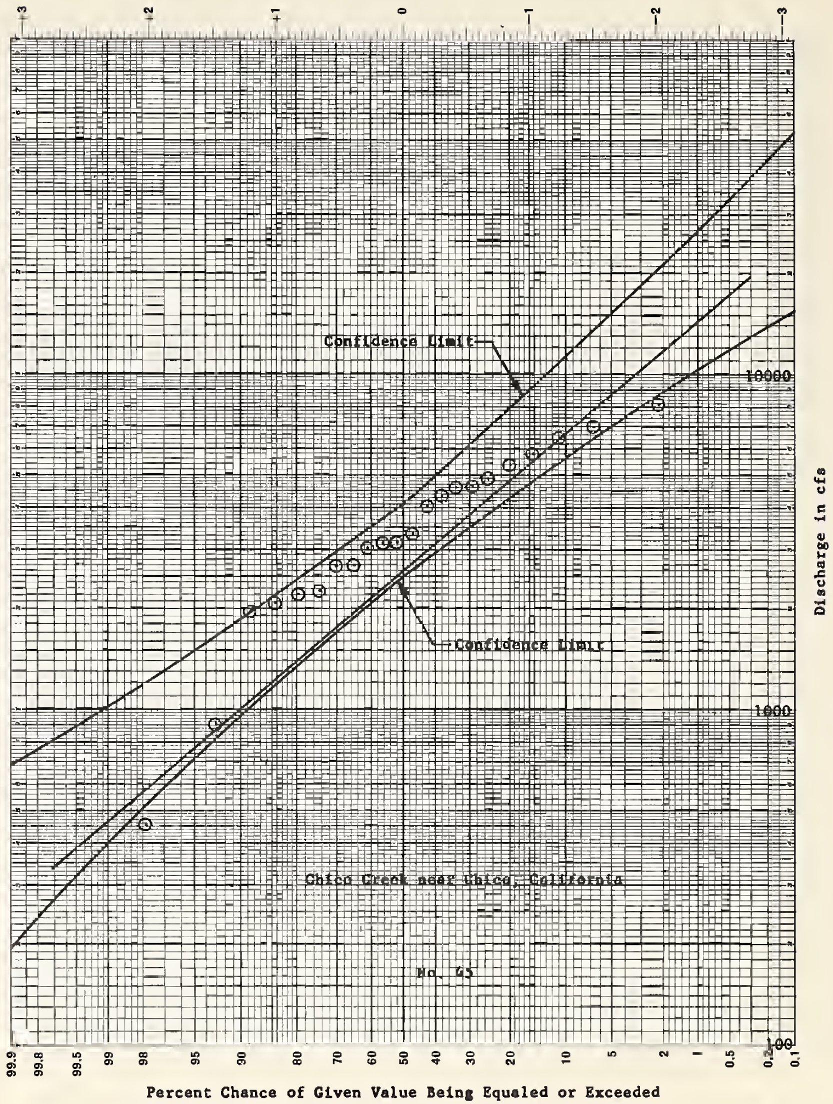

Discharge in cfs
10000
1000
Confidence Limit
Confidence Limit
Merced River at Happy Isles Bridge, California
No. 5
Percent Chance of Given Value Being Equaled or Exceeded.

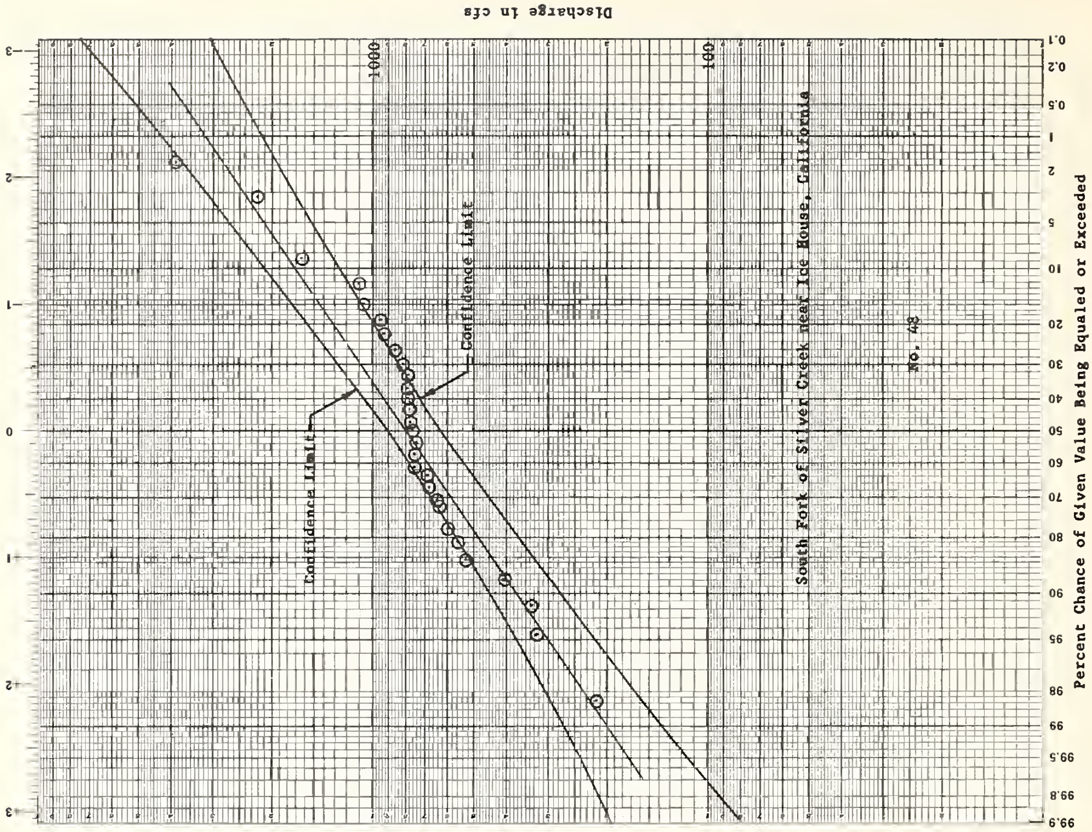

Discharge in cfs
1000
100
Confidence Limit
Confidence Limit
Plum Creek near Riverton, California
No. 8
Percent Chance of Given Value Being Equaled or Exceeded.

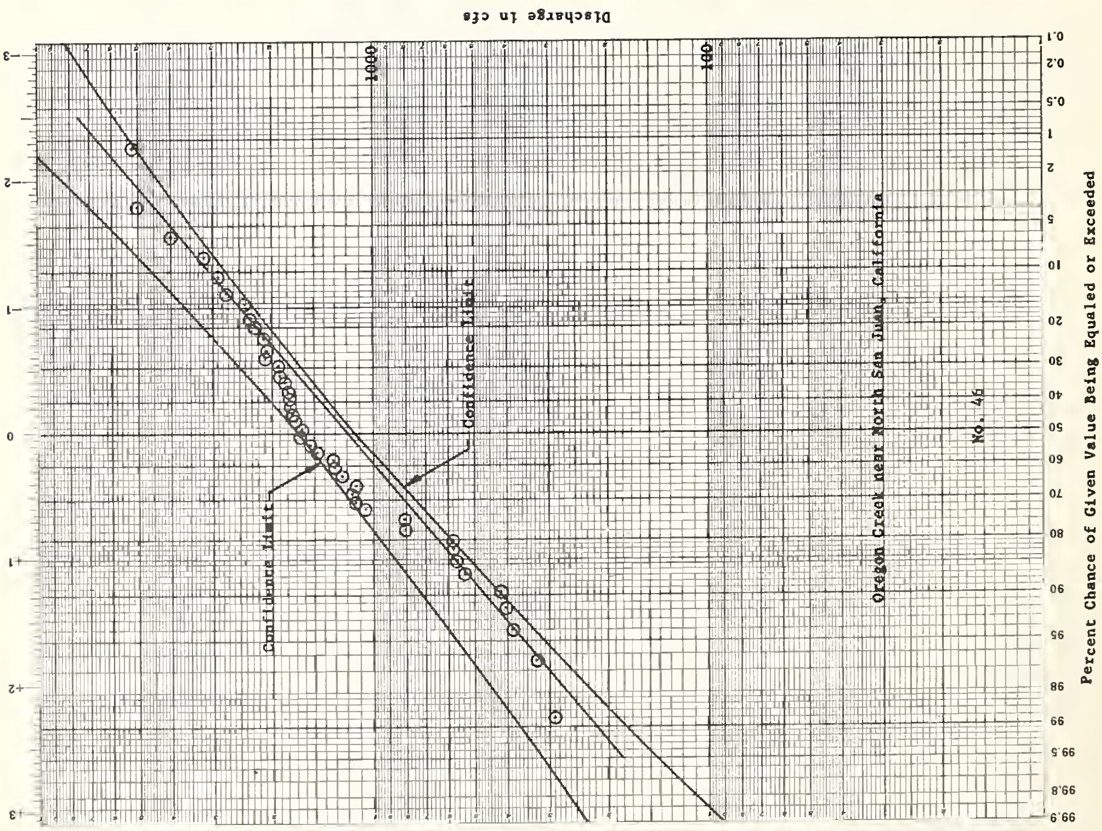

Discharge in cfs
1000
100
Confidence Limit
Confidence Limit
Mono Creek near Vermillion Valley, California
No. 16
Percent Chance of Given Value Being Equaled or Exceeded

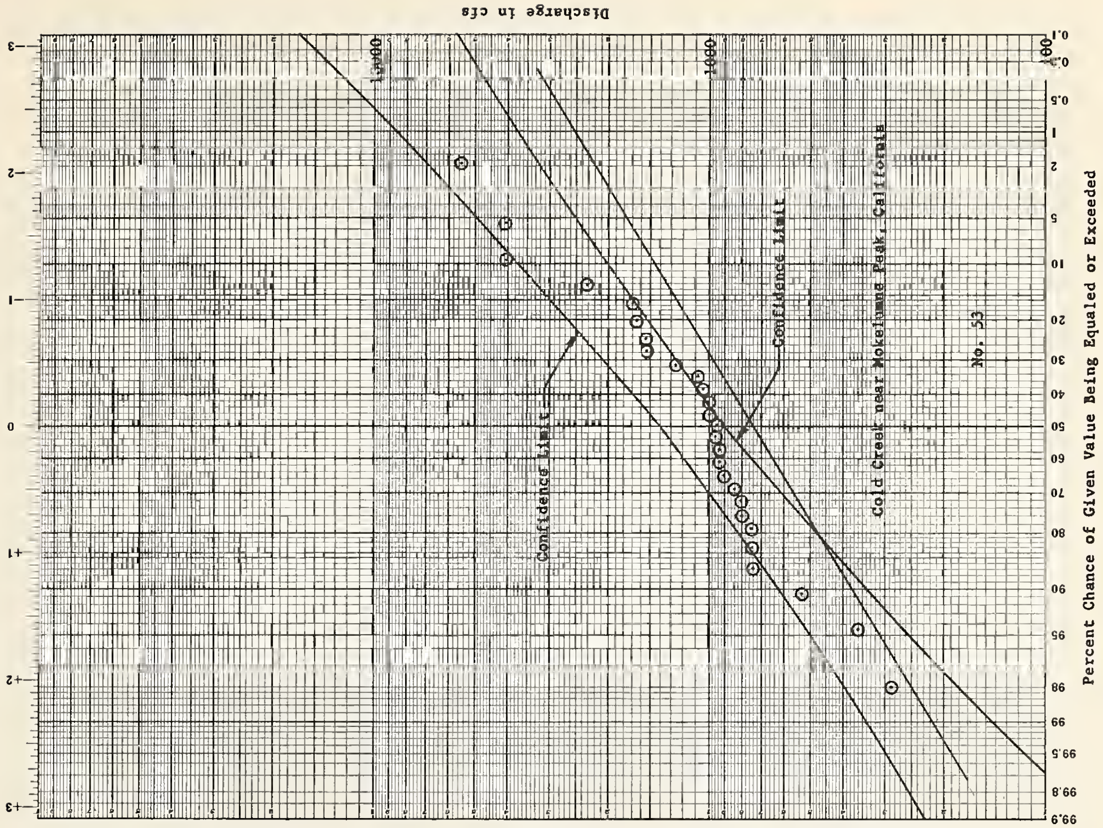

Discharge in cfs
10000
1000
100
Confidence Limit
Confidence Limit
North Fork of the Yuba River near Goodyear Bar, California
No. 11
Percent Chance of Given Value Being Equaled or Exceeded

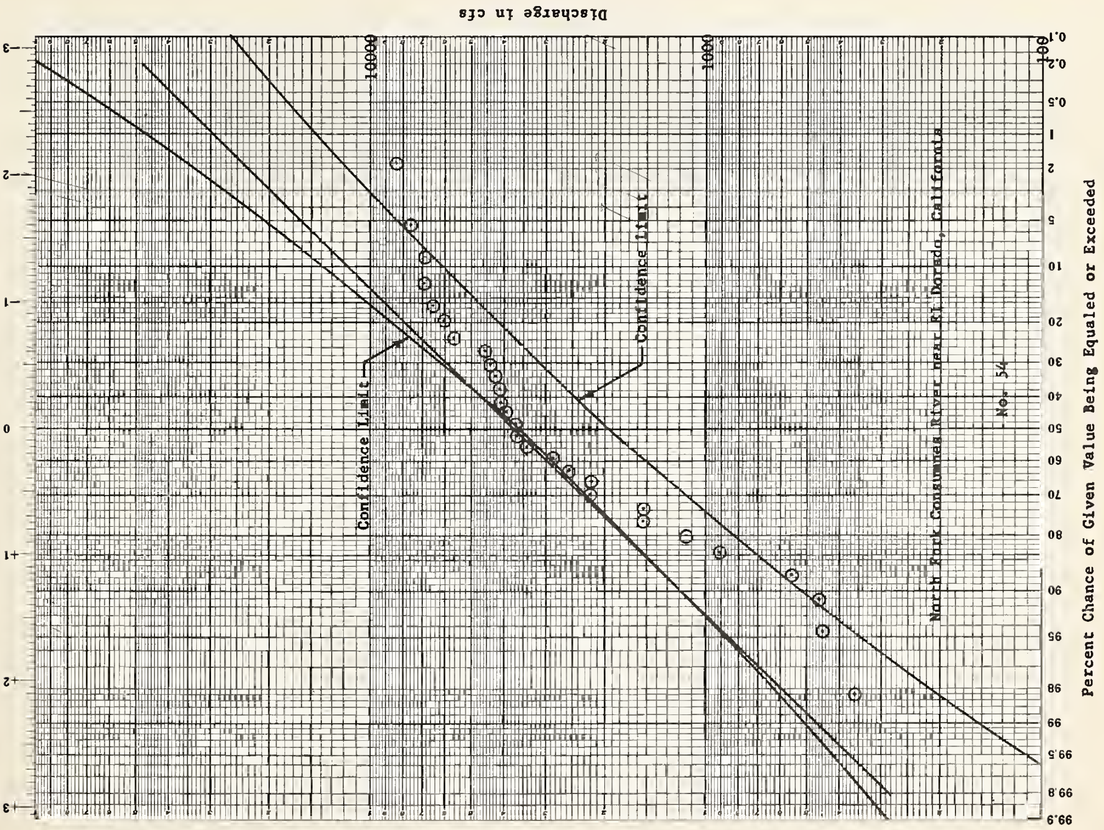

Confidence Limit
Confidence Limit
1000
100
Discharge in cfs
South Fork Mokelumne River near West Point, California
No. 44
Percent Chance of Given Value Being Equaled or Exceeded

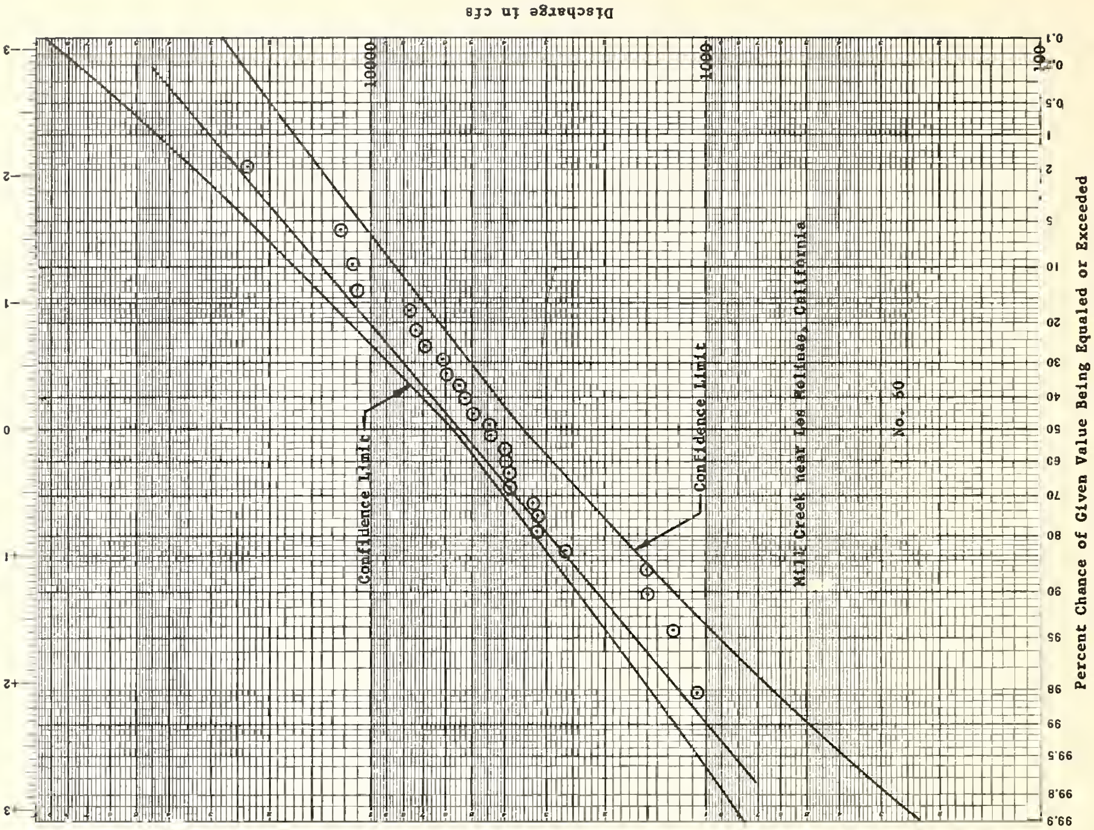

Confidence Limit
Confidence Limit
10000
1000
100
Discharge in cfs
Chico Creek near Chico, California
No. 45
Percent Chance of Given Value Being Equaled or Exceeded

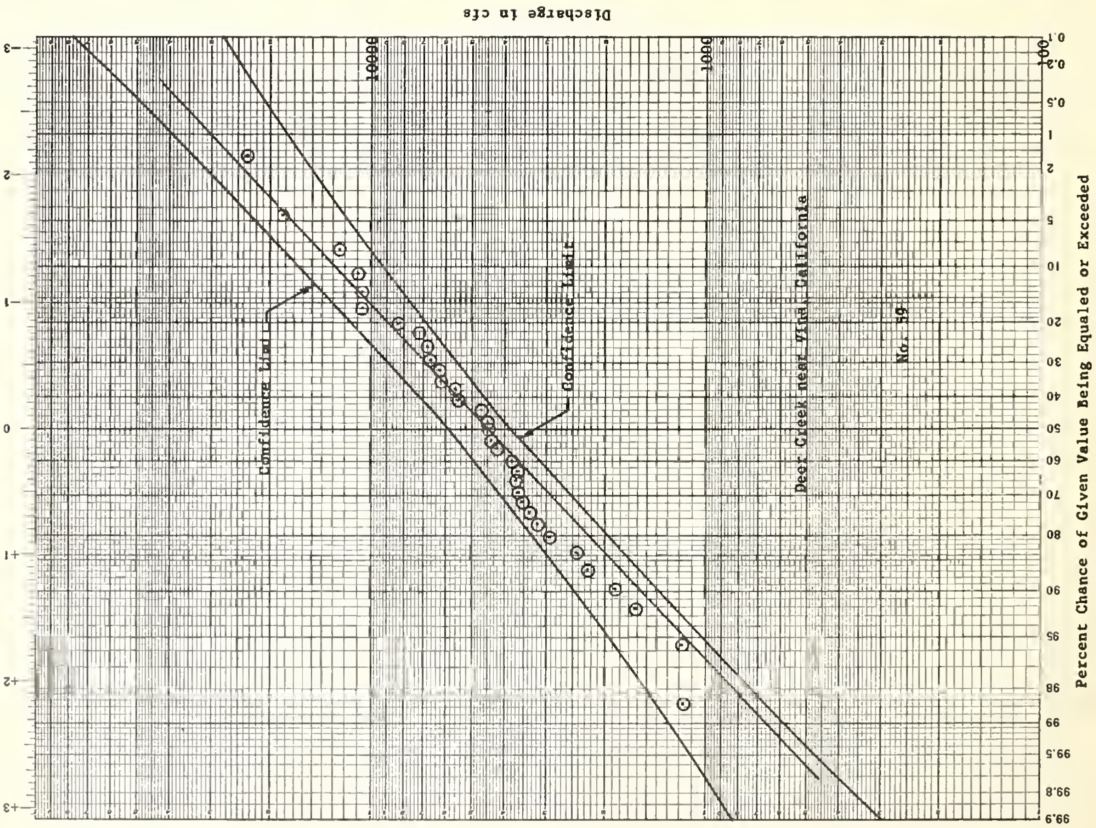

South Fork of Silver Creek near Ice House, California
No. 48
Confidence Limit
Confidence Limit
Discharge in cfs
Percent Chance of Given Value Being Equaled or Exceeded


Oregon Creek near North San Juan, California
No. 45
Confidence Limit
Confidence Limit
Discharge in cfs
Percent Chance of Given Value Being Equaled or Exceeded


Discharge in cfs
Percent Chance of Given Value Being Equaled or Exceeded
Cold Creek near Mokelumne Peak, California
No. 53
Confidence Limit
Confidence Limit


Discharge in cfs
Percent Chance of Given Value Being Equaled or Exceeded
North Fork Cosumnes River near El Dorado, California
No. 54
Confidence Limit
Confidence Limit


Discharge in cfs
100
1000
10000
Percent Chance of Given Value Being Equaled or Exceeded
Confidence Limit
Confidence Limit
Mill Creek near Los Molinos, California
No. 50


Discharge in cfs
100
1000
10000
Percent Chance of Given Value Being Equaled or Exceeded
Confidence Limit
Confidence Limit
Deer Creek near Vina, California
No. 59